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Possible Gauge Theoretic Origin for Quark-Lepton Complementarity

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Abstract

Similarity between the weak interaction properties of quarks and leptons has led to suggestions that the origin of lepton mixing angles may be related to those of quarks. In this paper, we present a gauge model based on $SU(2)_L \times SU(2)_R \times SU(4)_c$ group that leads to a new form for the quark lepton complementarity which predicts the solar neutrino mixing angle in terms of the Cabibbo angle for the case of inverted mass hierarchy for neutrinos. We also indicate how these ideas can be implemented in an E_6 inspired trinification $SU(3)_C \times SU(3)_L \times SU(3)_R$ model, which is more closely allied to string theory by the AdS/CFT correspondence.

I. INTRODUCTION

One of the major challenges for particle theory today is to understand the totally different pattern of mixings among neutrinos compared to quarks. Although many different ways have been proposed to tackle this challenge (for recent reviews, see [1]), no final consensus has emerged on what is the most promising way to proceed. Any new approach is therefore desirable.

In this paper, we will focus on understanding the solar mixing angle, $\theta_\odot \equiv \theta^\nu_{12}$, which is large but known not to be maximal. Since there are leptonic symmetries that lead to maximal solar mixing, a possible approach to this problem is to start in that limit and understand the deviation using new physics. There have been many attempts in the literature to achieve this goal [2]. In this note, we pursue a recent suggestion [3,4] according to which the deviation from maximality of the solar mixing may be related to the quark mixing angle $\theta_C \equiv \theta^q_{12}$. It is based on the observation that the mixing angle responsible for solar neutrino oscillations, $\theta_\odot \equiv \theta^\nu_{12}$ satisfies an interesting complementarity relation with the corresponding angle in the quark sector $\theta_{Cabibbo} \equiv \theta^q_{12}$ i.e. $\theta^\nu_{12} + \theta^q_{12} \simeq \pi/4$. This equation is satisfied to within a few per cent accuracy. While it is quite possible that this is purely accidental, it is interesting to pursue the possibility that there is a deep meaning behind it and see where it leads. We explore the possibility that this relation may be a reflection of a fundamental quark-lepton unification.

To see how quark lepton unification can possibly lead to a complementarity relation, first note that $\theta_\odot$ is the 12 entry of the PMNS matrix defined as $U_{PMNS} = U^\dagger_\ell U_\nu$ (where U_ℓ is the unitary matrix operating on the left handed charged leptons that diagonalizes the (e, μ, τ) mass matrix and U_ν is the corresponding one for the neutrino Majorana mass matrix) and θ_c is the corresponding entry in the CKM matrix defined as $V_{CKM} = U^\dagger_u U_d$ (where $U_{u,d}$ diagonalize the up and down quark mass matrices). Secondly suppose that the structure of neutrino and quark mass matrices at the high scale are such that to a leading order, the PMNS matrix is exact bimaximal [5] whereas the CKM matrix is an identity matrix and to next leading order we have the down quark and charged lepton mass matrices are equal (or very nearly so) in the basis where the up quark and neutrino Dirac masses remain unchanged, then a complementarity relation between quarks and leptons can emerge from quark-lepton unification at high scale.

In this paper, we make an attempt to connect the deviation of solar mixing angle from its maximal value to the Cabibbo angle using the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_c$ symmetry that unifies quarks and leptons [6] at high scale. We find that if we use the double seesaw formula for understanding small neutrino masses, then the departure of the solar angle from maximality is determined by the Cabibbo angle and the prediction for the solar mixing angle is consistent with experiments within present uncertainies. This model also predicts a "large" value for $\theta_{13} \simeq 0.15$.

We also indicate how these considerations may be extended to the case of a gauge group $[SU(3)]^3$ which unifies to the gauge group E_6 at high scale, that has quark-lepton unification.

II. GAUGE MODEL FOR QUARK-LEPTON COMPLEMENTARITY (QLC) IN THE 1-2 SECTOR

Before proceeding to the construction of the model, we first outline a set of conditions that are sufficient for obtaining a connection between the $\theta_\odot$ and θ_C. The full quark and lepton mass matrices should be written in a form such that, we have:

$$M_{u,d} \;=\; M^0_{u,d} + \delta M_{u,d} \tag{1}$$

$$M_\ell \;=\; M^0_\ell + \delta M_\ell \tag{2}$$

$$\mathcal{M}_\nu \;=\; U^*_{bm}\mathcal{M}^d_\nu U^\dagger_{bm} \tag{3}$$

where U_{bm} is the bimaximal PMNS matrix given by:

$$U_{bm} \;=\; \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \tag{4}$$

We require that $M^0_u \;=\; tan\beta M^0_d$ so that to leading order $V^0_{CKM} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix and $M^0_\ell = M^0_d$ due to quark lepton symmetry. We also choose $\delta M_u = 0$. Note also that we have chosen the $\delta M_\ell \approx \delta M_d$ at the scale where neutrino masses arise via seesaw mechanism i.e. exact quark-lepton symmetry to zeroth order but approximate quark-lepton symmetry upto the first order terms that generate the 12 mixing angles. The observed pattern in the V_{CKM} then arises from the matrix δM_d. Its similarity in form to δM_ℓ feeds this change to the lepton mixing matrix in a way such that a form of quark-lepton complementarity emerges in the end. The Cabibbo angle is small in our model due to it being related to the quark mass ratios. We also note that δM_d has to be different from δM_ℓ in order to fit the muon mass after correcting for extrapolation effects from the seesaw scale to the weak scale. The departure from maximality of solar mixing angle is predicted in terms of Cabibbo angle as well as muon and strange quark masses.

We first describe in detail how in a gauge theory based on the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_c$ the above conditions can be satisfied. In a subsequent section, we will indicate how the same considerations can be carried over to a trinification theory.

Let us start with the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_c$ with supersymmetry and an additional softly broken global symmetry on the theory given by $U(1)_X$ where $X = f_1 - f_2 - f_3$ with f_i representing the i-th family. In the lepton sector this corresponds to $L_e - L_\mu - L_\tau$ symmetry, which has been extensively discussed in the literature [7]. We make the usual assignment of fermions to the $\Psi(2,1,4) + \Psi^c(1,2,4^*)$ as follows:

$$\Psi \;=\; \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}_L \tag{5}$$

We add a set of three singlet fermions S_i and S^c_i to the theory (i=1,2,3 stands for generation) which also have the same fermion number as the normal matter multiplet. For the Higgs sector, we choose the following multiplets: $\phi \equiv (2,2,0)$; $\chi \equiv (2,1,4)$ and $\chi^c \equiv (1,2,4^*)$. In a supersymmetric theory, anomaly cancellation requires that we also have the fields: $\overline{\chi} \equiv (2,1,4^*)$ and $\overline{\chi^c} \equiv (1,2,4)$.

We break the gauge symmetry down to the standard model by the vevs of the fields $<\chi^c>=<\overline{\chi^c}>=v_R$. The standard model symmetry is then broken down by the vev $<\phi> \ = \begin{pmatrix}\kappa & 0\\ 0 & \kappa'\end{pmatrix}$. We keep $<\chi^0>=<\bar{\chi}>=0$, which can be a stable vacuum if we forbid the trilinear term $\chi\phi\chi^c$ by a discrete symmetry.

We can write down the Yukawa couplings of the model responsible for quark and lepton masses as follows:

$$W_Y \ = \ h_{ij}\Psi_i\phi\Psi^c_j + f_{ij}(\Psi_i\bar{\chi}S^c_j + \Psi^c_i\bar{\chi}^c S_j) \ + \ \mu_{ij}S_iS_j \tag{6}$$

We choose the Majorana mass for the S^c fields to be very heavy so that they decouple at TeV energies. This also decouples the $\Psi_i\bar{\chi}S^c_j$ term from the Lagrangian, which we wrote down above for completeness. Note that $U(1)_X$ symmetry restricts the form of μ_{ij} to be

$$\mu \ = \ \begin{pmatrix} 0 & \mu_1 & \mu_2\\ \mu_1 & 0 & 0\\ \mu_2 & 0 & 0\end{pmatrix} \tag{7}$$

After the $<\chi^c>$ and the $<\phi>$ vevs are turned on, we have the following zeroth order forms for the various fermion mass matrices:

$$\frac{\kappa'}{\kappa}M^0_u \equiv cot\beta M^0_u \ = \ M^0_d \ = \ M^0_\ell \tag{8}$$

where $M^0_u \ = \ \begin{pmatrix} m_{11} & 0 & 0\\ 0 & m_{22} & m_{23}\\ 0 & m_{23} & m_{33}\end{pmatrix}$ Since the up and down quark masses are proportional, the CKM matrix is the unit matrix. The leptonic mixing matrix is however not proportional to a unit matrix as can be seen below.

The mass matrix for (ν, ν^c, S) is given by:

$$M \ = \ \begin{pmatrix} 0 & M^0_u & 0\\ M^0_u & 0 & fv_R\\ 0 & f^Tv_R & \mu\end{pmatrix} \tag{9}$$

where M^0_u, fv_R are all 3×3 matrices whose forms are similar and the form of μ is given above. First point to note is that the matrix M^0_u can be diagonalized without effecting the form of μ. In general the matrix f can be written in the form $\begin{pmatrix} f_{11} & 0 & 0\\ 0 & f_{22} & f_{23}\\ 0 & f_{23} & f_{33}\end{pmatrix}$ due to the $U(1)_X$ symmetry.

The matrix in Eq. (9) is in the double seesaw form [8] which leads to small neutrino masses even with a multi-TeV scale for the seesaw. The scale however can also be close to the GUT scale[1]. In this note, we keep the double seesaw scale v_R near 100 TeV and include its effect (via renormalization group) on the fermion masses through a parameter Δ.

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[1]This possibility was pointed out by S. F. King (private communication).

Diagonalizing the full neutrino matrix in Eq.9 [8], one obtains the light neutrino mass matrix $\mathcal{M}_\nu$ to be

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_1 & m_2 \\ m_1 & 0 & 0 \\ m_2 & 0 & 0 \end{pmatrix} \tag{10}$$

where $m_1 = \frac{m_{11}m_{22}\mu_1}{f_{11}f_{22}v_R^2}$ and $m_2 = \frac{m_{11}\mu_1 m_{33}}{f_{11}\tilde{f}_{23}v_R^2} + \frac{m_{11}\mu_2 m_{33}}{f_{11}f_{33}v_R^2}$. Thus for instance, if $m_{ij} \sim$ GeV and $\mu_i \sim$ GeV and $v_R \simeq 100$ TeV, we get for neutrino masses $m_i \simeq 0.1$ eV, which is of the same order as the square root of the atmospheric neutrino mass difference square. We could increase v_R by appropriately adjusting the scale μ_{ij}. For instance, if v_R is assumed to be at the GUT scale, so would be μ_{ij}. Clearly, if $m_1 \simeq m_2$ we also get maximal mixing needed for atmospheric neutrino oscillation.

Note that solar mixing angle is maximal and in fact the PMNS matrix at this zeroth order level is bimaximal for $m_1 = m_2$ and the mass ordering of neutrinos is the so-called inverted type. Of course the model at the zeroth order level has the unpleasant feature that it predicts vanishing solar neutrino mass difference which however is corrected as we include higher order terms.

From the relation $M_u^0 = tan\beta M_d^0 = tan\beta M_\ell^0$, it may appear that, we have unacceptable relations among the masses of quarks and leptons. Models with such a property have been discussed before [9] and it has been shown how such models, if supersymmetric can be phenomenologically fully viable. These mass relations without further corrections imply that $m_s^0/m_b^0 = m_c/m_t$ and $m_\mu^0(M_c) = m_s^0(M_c)$, where M_c is the SU(4) scale. Note that m_s and m_μ cannot be close to each other at the multi-TeV scale since renormalization group extrapolation due to strong interactions increases m_s without changing m_μ much. Therefore, we will use the δM terms to get the right m_s/m_μ. We will see that we can get a sizable correction to the maximality of solar mixing angle dictated bt the zeroth order terms.

We will assume that the δM terms do not affect the third generation fermion masses. The ratio m_c/m_t then implies that $m_b = m_\tau \sim 10$ GeV. These masses are much higher than their observed values. It has however been noted that in supersymmetric theories, there are large gluino and Wino contributions to the bottom and tau masses, which can correct for this [9] and bring them down to observed values. For large $tan\beta$, there are two contributions to the quark masses and one for the charged lepton [10]:

$$\delta m_b = \frac{2\alpha_s m_{\tilde{g}}}{3\pi m_{\tilde{q}}^2}(m_b^0 \mu tan\beta + A_{33}^d m_0) + \frac{Y_t^2 \mu}{16\pi^2 m_{\tilde{q}}^2}(m_b^0 \mu + A_{33}^u m_0 tan\beta) \tag{11}$$

and

$$\delta m_\tau = \frac{g_1^2 m_{\tilde{B}}}{16\pi^2 m_{\tilde{l}}^2}(m_\tau^0 \mu tan\beta + A_{33}^e m_0) \tag{12}$$

Clearly, there are several independent parameters which one may adjust to get the bottom and the tau lepton mass right. Note that the quark mixings still vanish at this stage. As noted before, if we choose a value for $m_s \simeq m_\mu$ somewhere around 100 MeV. For v_R in the 100 TeV range, when we come down to the weak scale, m_s will increase and become bigger than m_μ. In the next section, we show how the higher order effects can fix this problem, generate quark and lepton mixing leading to quark-lepton complementarity as well as give the first generation masses right.

III. INCLUSION OF NEXT ORDER TERMS AND DERIVATION OF QUARK-LEPTON COMPLEMENTARITY

In order to obtain the CKM angles and the departure from bimaximal pattern for neutrino mixings, we need to include higher dimensional non-renormalizable terms in the theory that break $U(1)_X$. We assume that they contribute only to the down quark and charged lepton mass matrix in a quark-lepton symmetric way. For this purpose, in the 422-model, we include a $SU(2)_R$ triplet Higgs field δ_R which transforms under the gauge group as $(1,3,15)$ and a gauge singlet $\sigma(1,1,15)$, both of which have $X=+2$. We will also include a δ_L with zero vev so that it has no effect on the masses. This allows us to include two terms which can lead to violation of X charge after symmetry breaking:

$$\delta\mathcal{L} \;=\; \frac{1}{M}[f'_{ij}\psi_i^T\phi\vec{\tau}\cdot\vec{\delta}_R\psi_j^c + f''\psi_i^T\phi\sigma\psi_j^c] + \frac{1}{M^2}(\psi^T{}_1\phi\chi^c\bar{\chi^c}\psi_1^c) \tag{13}$$

where $i=1$ and $j=2,3$. As just noted, once $\vec{\delta}_R$ and σ acquire vevs, we can arrange their values and the couplings f' and f'' so that the contribution of these terms to the fermion mass appears predominantly in the down sector. Secondly due to the fact that the multiplets transform as **15** dim. representation of $SU(4)_c$, they will make different contribution to δM_d and δM_ℓ. We choose $<\delta_R>, <\sigma> \simeq <\chi^c>$ and $M \sim 10^2(<\delta_R>, <\sigma>)$ so that the magnitude of these terms of the right order of magnitude (i.e. 10^{-2} GeV for the first two and few $\times 10^{-3}$ GeV for the last term) so that we generate the right value for the Cabibbo angle. Clearly, the last term makes a small contribution (due to two powers of M in the denominators) to the electron mass but not big enough to contribute to quark masses. Collecting all these together and ignoring the third generation fermions we can write the mass matrices for the first two generation of quarks and leptons in the presence of the higher dimensional terms at the weak scale:

$$\begin{aligned} M_\ell &= \begin{pmatrix} m_0 & -3\delta' \\ -3\delta' & m_\mu^0 - 3\delta \end{pmatrix} \\ M_d &= \Delta\begin{pmatrix} 0 & \delta' \\ \delta' & m_\mu^0 + \delta \end{pmatrix} \end{aligned} \tag{14}$$

where m_μ^0 is the zeroth order contribution, δ and δ' are the next order contributions from Eq. (13) and Δ is renormalization effect on the quark masses from v_R down to M_Z. In this model $\theta_c \simeq \frac{\delta'}{m_\mu^0+\delta} \equiv \sqrt{\frac{m_d}{m_s}}$. On the other hand, the mixing angle for charged lepton is:

$$\theta^\ell \simeq \frac{-3\theta_C m_s}{\Delta m_\mu}. \tag{15}$$

We choose the scale v_R to be low (in the multi-TeV range) and choose for example of $m_s \sim m_\mu/2$ at v_R scale. The value of m_s increases somewhat as we move to the weak scale but still remains below m_μ[2]. For example if $\Delta = 1.5$ (corresponding to v_R in the multi-TeV range), we estimate $\theta^\ell \simeq \theta_C$. This leads to a prediction for the solar mixing angle [4] :

[2]Recent lattice calculations seem to prefer such low values

$$sin^2\theta_\odot \simeq 0.32 \tag{16}$$

The present 3σ fit to the solar and KamLand data gives $\sin^2\theta_\odot = 0.23 - 0.38$. It also leads to a prediction for the $\theta_{13} \simeq \frac{\theta_C}{\sqrt{2}} \simeq 0.15$, which can be tested in the next round of searches for this parameter.

In this model, we get $m_e \sim m_0 - \frac{9\theta_C^2 m_s^2}{\Delta^2 m_\mu}$ which for $m_s \simeq 75$ MeV is about $m_0 - 8$ MeV. We can adjust m_0 to get the right value of m_e.

The next issue we need to address is the origin of solar mass difference square. It can arise in a manner which does not affect the quark-lepton complementarity by adding a gauge singlet σ' with $X = 2$. This allows a term of the form $\lambda S_1 S_1 \sigma'$ with $\lambda \sim 10^{-2}$. This introduces a μ_{11} entry which the leads via double seesaw to the solar mass difference of the right order for $< \sigma' > \simeq \mu$.

IV. A POSSIBLE TRINIFICATION MODEL EXAMPLE

Another gauge model which can also provide a realization of QLC is the trinification model based on the gauge group $[SU(3)]^3$ [11] which arises from the grand unification group E_6. Here we assign the fermions to

$$\psi_i = \psi_R + \psi_L + \psi_c \equiv (3, 3^*, 1)_{Li} + (3^*, 1, 3)_{Li} + (1, 3, 3^*)_{Li} \tag{17}$$

Here the subscript (C,L,R) denotes the group under which the multiplet is a singlet. We then add three singlet fermions

$$S_i = (1, 1, 1)_i \tag{18}$$

For scalars, we use

$$\Phi = \phi_c + \phi_L + \phi_R \tag{19}$$

where $\phi_c \sim (1, 3, 3^*) + c.c.$, $\phi_R \sim (3, 3^*, 1) + c.c.$, and $\phi_L \sim (3, 1, 3^*) + c.c.$

Again we impose a $X = f_1 - f_2 - f_3$ symmetry. In addition before symmetry breaking there is an $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_3$ quasi-simple symmetry where the Z_3 interchanges the $SU(3)_i$. Here the $SU(3)_L$ plays a role similar to the generalized weak isospin of the 331-model [12]. However, it is necessary to respect the fully unified simple E_6 symmetry to obtain the required relation between quark and lepton couplings.

There are two types of Yukawa couplings possible in this theory: for quarks the Yukawa responsible giving mass are of the form $\psi_L \psi_R \phi_c$ whereas for leptons it has the form $\epsilon^{abc}\epsilon_{pqr}\psi^p_{c,a}\psi^q_{c,b}\phi^r_{c,c}$. At the $[SU(3)]^3$ level, these two couplings are independent and therefore there is no connection between quarks and leptons. However, if the theory is assumed to emerge from an E_6 GUT model, the two couplings arise from the same $[27]^3$ coupling and become equal. In order to derive QLC relations, we need to assume this. The leptonic part of the Yukawa interaction can then be written as (i, j, k are generation indices):

$$\mathcal{L}_Y = h_{ij}\psi_{c,i}\psi_{c,j}\phi_c + f_{ij}\psi_{c,i}S_{Lj}\phi_c^* + \mu_{ij}S_{Li}S_{Lj} \tag{20}$$

In this 333-model, the VEVs break the Z_3 symmetry. The $< \phi_c >$ has the form

$$< \phi_c > \; = \; \begin{pmatrix} v_u & 0 & 0 \\ 0 & v_d & v_R \\ 0 & v_L & M_3 \end{pmatrix}. \tag{21}$$

There are two kinds of vev's in $< \phi >$: the $v_{u,d,L}$ are of order of the weak scale whereas v_R and M_3 are in the multi-TeV range and they break $SU(3)_R$. These mass matrices lead to the double seesaw neutrino mass matrix given above. The rest of the considerations are similar to the previous model and we do not go into the details here.

This $SU(3)^3 \times Z_3$ model has the advantage that it is obtainable as a quiver theory [13] from superstring theory by using the AdS/CFT correspondence [14].

V. DISCUSSION

In this paper, we have presented two possible gauge models that realize the quark-lepton complementarity in the 12-sector. An essential ingredient in our first approach is the Pati-Salam $SU(4)_c$ gauge symmetry which has been suspected in ref. [4] to be a possible way to obtain the QLC relation. For the first time, we present explicit models that realize this relation. As we have noted in the body of the paper, the constraints required to obtain the QLC relation poses highly nontrivial challenges for model building. What we show is that it is possible to have a natural realization under a plausible set of assumptions. The double seesaw appears to be crucial in our discussion as is the inverted mass spectrum for neutrinos that is a consequence of $L_e - L_\mu - L_\tau$ symmetry. The model predicts a value of θ_{13} not far below its present upper limit.

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